

## Supplementary Information

# Second harmonic generation of laser beams in transverse mode locking states

ZiLong Zhang,<sup>a,b,c,\*</sup> Yuan Gao,<sup>a,b,c</sup> XiangJia Li,<sup>d</sup> Xin Wang,<sup>a,b,c</sup> SuYi Zhao,<sup>a,b,c</sup> Qiang Liu,<sup>e,f</sup> ChangMing Zhao<sup>a,b,c</sup>

<sup>a</sup> School of Optics and Photonics, Beijing Institute of Technology, 5 South Zhongguancun Street, 100081 Beijing, China

<sup>b</sup> Key Laboratory of Photoelectronic Imaging Technology and System, Ministry of Education of People's Republic of China

<sup>c</sup> Key Laboratory of Photonics Information Technology, Ministry of Industry and Information Technology

<sup>d</sup> Department of Aerospace and Mechanical Engineering, School for Engineering of Matter, Transport and Energy, Arizona State University, 501 E. Tyler Mall, Tempe, AZ 85287

<sup>e</sup> Key Laboratory of Photonic Control Technology (Tsinghua University), Ministry of Education, Beijing 100084, China

<sup>f</sup> State Key Laboratory of Precision Measurement Technology and Instruments, Department of Precision Instrument, Tsinghua University, Beijing 100084, China

E-mail: zlzhang@bit.edu.cn

The spectrum of a fundamental frequency beam (FFM) in transverse mode locking (TML) states detected by the photodetector and recorded by a radio frequency (RF) spectrum analyzer is shown in Fig. S1 (a). There was no mode beat can be detected. While, if the two modes were no locked, there is a spectrum can be measured. And the value of the mode beat frequency varies with the order of the two basic modes from tens of MHz to 100 MHz.

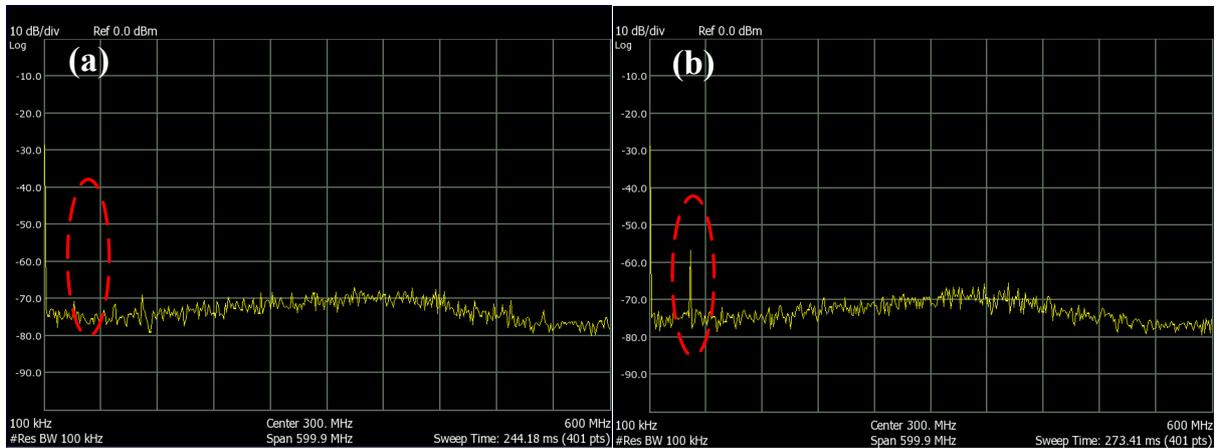


Figure. S1 The RF spectra of the TML mode and the corresponding unlocked states. (a) is the measured spectrum of the TML mode. And (b) is the unlocked states with a mode beat be measured.

Except the detailed experimental and simulated results in Fig.5, some more experimentally achieved far field SHG beam patterns are also presented in Fig.S2. By comparing with Fig.2, one can see that the patterns in Fig.S2 (a) to (c) are in coincident with that in row III of Fig.3 (b), row VI of Fig.3 (f), and row VII of Fig.3 (g), respectively. And Fig.S2 (d) to (f) show some far field SHG patterns generated by TML FFM composed of higher order ( $m+n=4$  or  $5$ ) basic HG or LG modes.

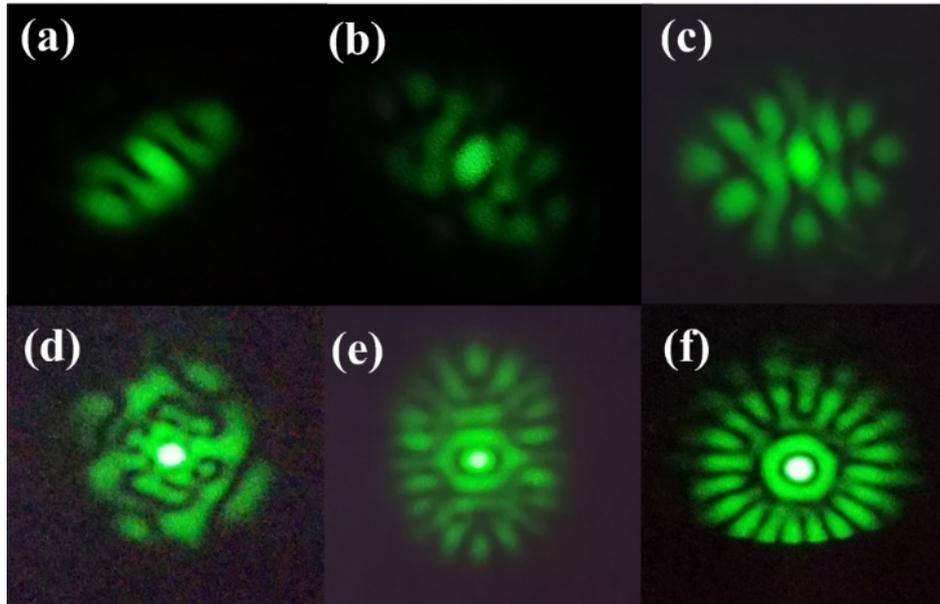


Figure.S2 Some more far field patterns of the SHG beams.

The beam pattern will experience a quick variation after a new adjustment of the pumping location and before the thermal equilibrium inside the cavity. When temperature inside the cavity is stable, the beam pattern will be in a relatively stable condition. For most SHG beam patterns shown in the manuscript can be maintained for a few minutes when the thermal condition is slowly changed under the current temperature controlling setups. For example, we can achieve a really stable SHG beam pattern with low order locked fundamental frequency transverse modes, such as  $HG_{02}+HG_{20}\exp(i3/4\pi)$ ,  $LG_{0,2}+LG_{0,-2}$  and  $LG_{0,3}+LG_{0,-3}$ , for about tens of minutes. Based on our experimental results, the SHG of the TML mode is promising to operate as or close to that of a TEM00 mode if the engineering work of thermal controlling is well done.

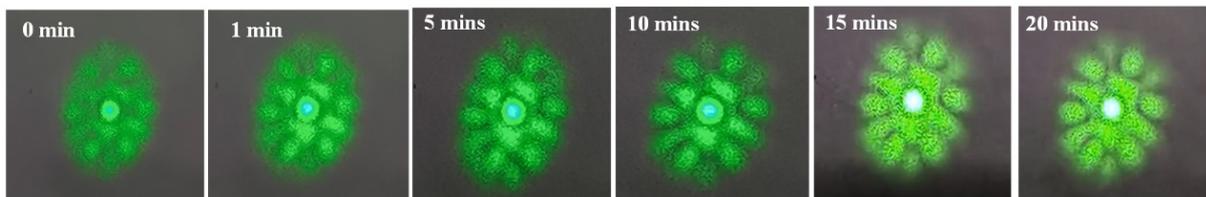


Figure. S3 The above figures show the beam patterns of  $LG_{0,3}+LG_{0,-3}$  mode in 20 mins measurement. In the first 10 mins, we completely recorded the patterns by normal camera video, and in the later 10 mins it was recorded by the method of time-lapse photography. After about 20 mins, we made a slight adjustment on the pump beam to break the stable state of the beam pattern and to reset a new pattern.